

## Mathematics Test Explanations

1. **The correct answer is C.** To find the total distance in miles that Shannon walked, add  $1\frac{2}{3}$  and  $2\frac{3}{5}$ . To add mixed numbers, find the least common denominator. The least common denominator of 3 and 5 is  $3 \times 5$ , or 15. To convert  $\frac{2}{3}$ , multiply by  $\frac{5}{5}$  (*hint:  $\frac{5}{5} = 1$ , and multiplication by 1 does not change the value of a number*). The result is  $\frac{10}{15}$ . To convert  $\frac{3}{5}$ , multiply by  $\frac{3}{3}$ . The result is  $\frac{9}{15}$ . To add  $1\frac{10}{15}$  and  $2\frac{9}{15}$ , first add 1 and 2 and then  $\frac{10}{15}$  and  $\frac{9}{15}$ . The result is  $3\frac{19}{15}$ , which reduces to  $4\frac{4}{15}$ .

Answer choice A is the most popular incorrect answer and comes from adding the whole numbers and then adding the numerators and the denominators separately.

2. **The correct answer is J.** To find an equivalent expression, multiply the constants ( $4 \times 3 \times 2 = 24$ ), combine the  $x$  terms ( $x^3 \times x \times x \rightarrow x^{3+1+1} \rightarrow x^5$ ), and combine the  $y$  terms ( $y^2 \times y^2 \rightarrow y^{2+2} \rightarrow y^4$ ). The result is  $24x^5y^4$ .

The most common incorrect answers are F and H, which come from multiplying the exponents of the  $x$  and  $y$  terms instead of adding them. If you chose G, you probably added the constants instead of multiplying them.

3. **The correct answer is C.** To find Mr. Wilk's pay per day, divide his annual salary, \$33,660, by the total number of days he works, 180. His pay per day is  $\frac{33,660}{180}$ , or \$187. When Mr. Wilk takes a day off without pay and the school pays a substitute \$85, the school district saves the difference in these amounts,  $187 - 85$ , or \$102.

Answer choice E, the most common incorrect answer, is simply Mr. Wilk's pay per day and not the difference between his pay and a substitute's pay.

4. **The correct answer is G.** To find the score on the fifth 100-point test that will yield an average score of 80, first calculate the total of the four scores already obtained:  $63 + 72 + 88 + 91 = 314$ . To obtain an average of 80 on 5 tests, the total score of all 5 tests must be  $80 \times 5$ , or 400. The score needed on the last test is equivalent to  $400 - 314$ , or 86.

Answer choice A is the average of the 4 scores, rounded to the nearest whole point.

5. **The correct answer is B.** To find the oxygen saturation level, divide the current number of milligrams per liter by the capacity milligrams per liter:  $\frac{6.4}{9.5}$ . Convert the result (0.6737) into a percent by multiplying by 100: 67.37% is approximately equal to 67%.

6. **The correct answer is H.** To find the length of fence needed to surround a rectangular lot 125 feet by 185 feet, calculate the perimeter. The formula for perimeter of a rectangle is 2 times the sum of the length and width, or  $P = 2(l + w)$ . Calculate the perimeter as follows:  $2(125 + 185) = 2(310)$ , or 620.

7. **The correct answer is C.** To find an equivalent expression, simply distribute the  $a$ , as follows:  $ab - ac + ad$ . Remember to keep track of the negative sign.

8. **The correct answer is G.** To solve for  $x$  in the equation  $6x - 3 = -5x + 7$ , add  $5x$  and 3 to both sides of the equation, which results in the equation  $11x = 10$ . Divide both sides by 11, which results in  $x = \frac{10}{11}$ .

9. **The correct answer is B.** These four numbers will form an arithmetic sequence, a sequence in which each pair of successive terms differs by the same number. To find the difference, define  $d$  as that difference, 13 as the first term, and 34 as the fourth term. By definition, the second term is  $13 + d$ . The fourth term, 34, can also be written as  $(13 + d + d) + d$ . Using that expression, obtain the equation  $34 = 13 + d + d + d$ , or  $34 = 13 + 3d$ . After subtracting 13 from both sides, divide by 3, which results in  $7 = d$ . The difference is 7. Thus the second term is  $13 + 7$ , or 20, and the third term is  $20 + 7$ , or 27.

10. **The correct answer is J.** To calculate the value of  $x^2 + \sqrt{x}$ , first solve  $x^3 = 729$  for  $x$ . The solution is the cube root of 729, which is 9. Substitute 9 into the original expression, arriving at  $9^2 + \sqrt{9}$ . This expression simplifies to  $81 + 3$ , or 84.

11. **The correct answer is C.** To find the volume, substitute  $\frac{4}{3}$  for  $r$  in the equation  $V = \left(\frac{4}{3}\right)\pi r^3$  as follows:

$$\left(\frac{4}{3}\right)\pi\left(\frac{4}{3}\right)^3$$

$$= \left(\frac{4}{3}\right)\pi\left(\frac{64}{27}\right)$$

$$= \left(\frac{256}{81}\right)\pi$$

Recall that  $\pi = \text{approx. } 3.14$ , so  $\left(\frac{256}{81}\right)(3.14)$  is about 9.92, or 10 when rounded to the nearest cubic inch.

- 12. The correct answer is F.** The probability that the gumball chosen will NOT be green when there are 6 yellow gumballs, 5 green gumballs, and 4 red gumballs is the number of favorable outcomes (the number of times a yellow or red gumball can be chosen) divided by the number of total outcomes (the total number of gumballs). The number of favorable outcomes is 10 because there are 6 yellow gumballs and 4 red gumballs. The total number of outcomes is  $6 + 5 + 4$ , or 15. Thus the probability of the gumball NOT being green is  $\frac{10}{15}$ , which can be reduced to  $\frac{2}{3}$ .

Answer choice G is incorrect because it is the probability that a chosen gumball *will* be green.

- 13. The correct answer is D.** To find the number of sports awards earned, multiply the number of participants in each sport by the ratio for that sport, and then add these 4 products. This is a matrix multiplication, as shown below:

$$[25 \ 30 \ 50 \ 80] \begin{bmatrix} 0.2 \\ 0.5 \\ 0.3 \\ 0.4 \end{bmatrix}$$

$$= 25(0.2) + 30(0.5) + 50(0.3) + 80(0.4)$$

$$= 5 + 15 + 15 + 32 = 67$$

- 14. The correct answer is G.** To find the average number of students per section enrolled in US History, find the total number of students in all sections and divide by the number of sections. Add  $25 + 29 + 24$  to get 78, then divide by 3. This results in an average of 26 students enrolled per section in US History.

If you selected answer choice F, you found the median, or middle number (which is not always the average), of 24, 25, and 29.

- 15. The correct answer is B.** The total number of books available is  $(30 - 3) + (30 - 5)$ , or  $27 + 25$ , which is 52. To find the class periods for which there are not enough books, find the total number

## PRACTICE TEST 1 ANSWERS AND EXPLANATIONS

of books needed for each period, as given in the table below.

PERIOD	1	2	3	4	5
BOOKS NEEDED	23	49	56	50	27

The only entry in the table with more than 52 is 56 for period 3.

If you selected answer choice E, you probably used 60 as the number of available books and did not take into account the 8 missing books.

- 16. The correct answer is G.** Because the sum of each row is equivalent, the sum of row 1 is the same as the sum of row 2.

$$\text{Row 1: } (-4x) + 9x + 2x = 7x$$

$$\text{Row 2: } 7x + ? + (-3x) = 4x + ?$$

The question mark must represent  $3x$ , because  $7x = 4x + 3x$ . You could also perform these calculations using the sum values in column 1 and column 2.

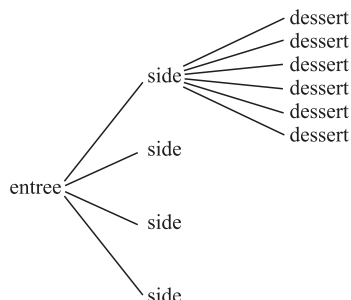
If you selected answer choice K, you may have thought that each sum must be 0 and found that  $-4x$  would make the sums of row 2 and column 2 equal 0.

- 17. The correct answer is E.** The  $x$ -coordinate is positive if A is to the right of the  $y$ -axis. The  $y$ -coordinate is positive if  $y$  is above the  $x$ -axis. The table below shows the sign of  $x$  and the sign of  $y$  in the four quadrants.

QUADRANT	SIGN OF	
	$x$	$y$
I	+	+
II	-	+
III	-	-
IV	+	-

Thus the signs are the same in Quadrants I and III only.

- 18. The correct answer is J.** To find the number of distinct complete meals that Reggie can make from 5 different entrees, 4 different sides, and 6 different desserts, multiply the quantities in the 3 different groups together. Thus, there are  $(5)(4)(6)$ , or 120 distinct meals that Reggie can make. The figure below shows that for each meal, there are 4 sides, and for each side there are 6 desserts.



19. **The correct answer is C.** To find the number of liters of carbonated water needed to produce 750 bottles of soda, set up a proportion with ratios of liters of carbonated water to bottles of soda, as follows:

$$\frac{10,000}{3,000} = \frac{x(\text{liters carbonated water})}{750}$$

Cross-multiply and solve for  $x$ .

$$3,000x = 7,500,000$$

$$x = 2,500$$

20. **The correct answer is F.** To find the length of the diagonal, apply the Pythagorean Theorem; the sides of the rectangle are the legs of a right triangle and the diagonal of the rectangle is the hypotenuse of the right triangle. Thus  $c^2 = 20^2 + 48^2$ , and  $c = 52$ .
21. **The correct answer is A.** To find an equivalent expression for  $\frac{a}{c}$ , either multiply or divide both the numerator and denominator by the same value. Because the question asks for all positive integers  $a$ ,  $b$ , and  $c$ , and you are looking for an expression that is equivalent to  $\frac{a}{c}$ , multiply  $\frac{a}{c}$  by  $\frac{b}{b}$  to get  $\frac{(a \times b)}{(c \times b)}$ , answer choice A.
22. **The correct answer is H.** The slope-intercept form of the equation of a line states that  $y = mx + b$ . To find the slope-intercept form of the equation  $6x - 2y - 4 = 0$ , you must isolate  $y$  on the left side of the equation, as follows:

$$6x - 2y - 4 = 0$$

$$-2y - 4 = -6x$$

$$-2y = -6x + 4$$

$$y = 3x - 2$$

If you selected answer choice J, you probably forgot to switch the signs when dividing by  $-2$ .

It is crucial to multiply all terms on both sides of the equation to arrive at a correct answer.

23. **The correct answer is E.** To solve the quadratic equation  $x^2 + 25x = 0$  for  $x$ , factor out an  $x$  on the left side of the equation:  $x(x + 25)$ . Now, apply the zero product rule:  $x = 0$  or  $x + 25 = 0$ . If  $x + 25 = 0$ , then  $x = -25$ , which is answer choice E.
24. **The correct answer is J.** To find  $\tan B$  in  $\triangle ABC$ , take the ratio of the length of the opposite side to the length of the adjacent side:  $AC$  to  $BC = c$  to  $a$ , or  $\frac{c}{a}$ .  
Answer choice F is  $\cos B$ ; answer choice G is  $\cot B$ ; answer choice H is  $\sec B$ ; answer choice K is  $\sin B$ .
25. **The correct answer is C.** To find the radius, use the right triangle shown in the diagram. Half of the length of the chord is 4 inches, which is the length of one leg. The other leg is 3 inches long, and the hypotenuse is  $r$  inches long. (Note: this is a right triangle because the distance between a point and a line is measured perpendicular to the line.) Use the Pythagorean Theorem, as follows:  $r^2 = 3^2 + 4^2 \rightarrow r^2 = 9 + 16 \rightarrow r^2 = 25 \rightarrow r = 5$  inches.  
If you selected answer choice E, you probably used 8 and 3 for the leg lengths and got  $r^2 = 73$ , which makes  $r$  equivalent to about 8.5 inches.
26. **The correct answer is K.** To find the force  $F$  (in newtons) corresponding to the spring length,  $L$ , of 0.23 meters when the relationship is given by the equation  $L = \left(\frac{2}{3}\right)F + 0.05$ , first substitute 0.23 for  $L$  to get  $0.23 = \left(\frac{2}{3}\right)F + 0.05$ . Next, subtract 0.05 from both sides to get  $0.18 = \left(\frac{2}{3}\right)F$ . Finally, multiply by  $\left(\frac{3}{2}\right)$ , since dividing by a fraction is equal to multiplying by its reciprocal, to arrive at  $0.27 = F$ .
27. **The correct answer is C.** To find the uniform depth, use the formula for volume,  $V$ , of a rectangular prism with the height  $h$ , length  $l$ , and width  $w$ ,  $V = (l)(w)(h)$ . Substitute the given values for the variables and solve for  $h$ :  $12,000 = (62)(85)(h)$ , or  $12,000 = 5,270h$ . Thus  $h = \frac{12,000}{5,270}$ , or about 2.277, which is between 2 and 3.
28. **The correct answer is G.** To find the length of the segment  $LM$  in  $\triangle LMN$ , where the length of the hypotenuse is 22 and the cosine of angle  $L$  is

$\frac{3}{4}$ , use the definition of cosine, which is the ratio of the length of the adjacent side to the length of the hypotenuse. In  $\triangle LMN$ , the cosine of angle  $L$  is the ratio of the length of segment  $LM$  to the length of the hypotenuse. Substitute the length of the hypotenuse and solve for  $LM$ , as follows:

$$\frac{3}{4} = \frac{LM}{22}$$

$$4 \times LM = 22 \times 3$$

$$LM = \frac{66}{4}, \text{ or } 16.5, \text{ answer choice G.}$$

- 29. The correct answer is A.** To find the fraction of apples grown in Appleton, divide the number of apples grown in Appleton by the total number of apples grown. The table below shows the conversion of apple symbols to numbers for the 4 cities, as well as the total number of apples grown.

CITY	NUMBER OF APPLES GROWN
Golden Hills	4,500
Red Falls	3,000
Appleton	2,500
Shady Acres	2,000
All Cities	12,000

The fraction of apples grown in Appleton is  $\frac{2,500}{12,000}$ , or  $\frac{5}{24}$ .

If you selected answer choice D, the most common incorrect answer, you probably used the number grown in Appleton divided by the total number of apples from the other 3 towns only.

- 30. The correct answer is H.** You are given that the length of  $AC$  is 19 units and the length of  $BD$  is 14 units. In addition, points are along segment  $AD$  as shown in the problem. Segment  $BC$  is the intersection of segment  $AC$  and segment  $BD$ . Therefore, the sum of the lengths  $AC$  and  $BD$  is the same as the sum of the lengths  $AD$  and  $BC$ . Substitute the actual lengths in  $AC + BD = AD + BC$  as follows:  $19 + 14 = 25 + BC \rightarrow 33 = 25 + BC \rightarrow 8 = BC$ .

- 31. The correct answer is D.** To find the  $x$ -coordinate where the lines with equations  $y = -2x + 7$  and

$y = 3x - 3$  intersect, set  $-2x + 7$  equal to  $3x - 3$  and solve for  $x$ :

$$-2x + 7 = 3x - 3$$

$$-5x + 7 = -3$$

$$-5x = -10$$

$$x = 2$$

- 32. The correct answer is K.** To solve the equation  $S = 4T - 7$  for  $T$ , add 7 to both sides to get  $S + 7 = 4T$ , and divide by 4 on both sides to get  $\frac{(S + 7)}{4}$ .

- 33. The correct answer is D.** The area for a parallelogram with base  $b$  and corresponding height  $h$  is  $(b)(h)$ . For parallelogram  $ABCD$ , segment  $AD$  is the base, with length  $5 + 15$ , or 20 inches, and the corresponding height is 12 inches. Therefore, the area is  $(20)(12)$ , or 240 square inches.

The most common incorrect answer is E, which is the result of multiplying the two side lengths:  $(5 + 15)(13) = 20(13)$ , or 260.

- 34. The correct answer is F.** To find  $(a - b)^4$  given  $b = a + 3$ , substitute  $a + 3$  for  $b$ , as follows:

$$(a - (a + 3))^4$$

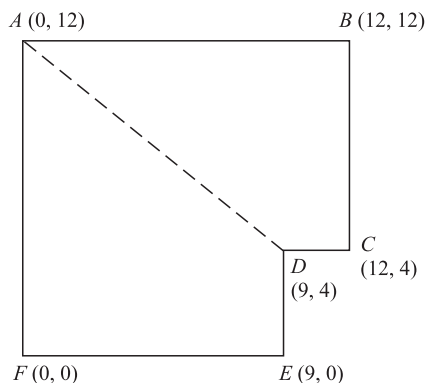
$$= (a - a - 3)^4$$

$$= (-3)^4, \text{ or } 81.$$

If you get stuck on this one, you can try choosing a specific value for  $a$ , such as 2. Then  $b = 5$  and  $(a - b)^4 = (2 - 5)^4 = 81$ .

If you selected answer choice K, you might have gotten  $-3$  for  $(a - b)$ , but solved  $-(3^4)$  instead of  $(-3)^4$ , thus arriving at an answer of  $-81$ . Remember that when you have an even numbered exponent, you can eliminate negative answer choices.

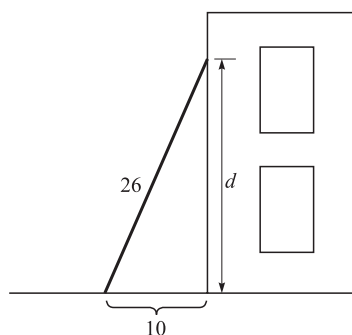
- 35. The correct answer is B.** To find the location of the park office located halfway between points  $A$  and  $D$ , it makes sense to give coordinates to the points in relation to an origin (see diagram below). In this case it makes sense to choose point  $F$  as the origin because it is in the bottom left of the figure. The first coordinate is the number of miles east of the origin, and the second coordinate is the number of miles north of the origin.



The park office is at the midpoint of the segment  $AD$ , and so the midpoint formula applies. For points with coordinates  $(x_1, y_1)$  and  $(x_2, y_2)$ , the midpoint has coordinates  $\left[\frac{(x_1 + x_2)}{2}, \frac{(y_1 + y_2)}{2}\right]$ . For  $A(0, 12)$  and  $D(9, 4)$ , the midpoint is  $\left(\frac{[0 + 9]}{2}, \frac{[12 + 4]}{2}\right)$ , or  $\left(\frac{9}{2}, 8\right)$ . However, the problem asks you to relate the location of the office to its distance and direction from point  $A$ . To do so, subtract the coordinates of point  $A$  from the coordinates of the midpoint:  $\left(\frac{9}{2} - 0, 8 - 12\right)$ , or  $\left(\frac{9}{2}, -4\right)$ . Thus, the location of the office relative to point  $A$  is  $4\frac{1}{2}$  miles east and 4 miles south.

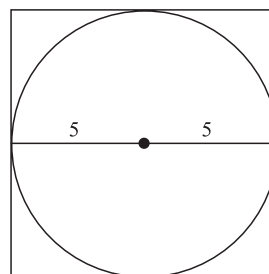
- 36. The correct answer is J.** A simple way to solve this problem is to let the larger number be  $y$ . Therefore, you know that  $y = 3x + 4$ , and that  $2y + 4x = 58$ . Substitute  $3x + 4$  for  $y$  in the last equation to arrive at  $2(3x + 4) + 4x = 58$ . This equation allows you to solve for  $x$ .

- 37. The correct answer is E.** To find out how far a 26-foot ladder reaches up a building when the base of the ladder is 10 feet away from the building, it is useful to draw a picture, as shown below:



As you can see, the ladder forms the hypotenuse of a right triangle with a length of 26, and the base of the ladder is 10 feet away from the building. Using the Pythagorean Theorem,  $26^2 = 10^2 + d^2$ , where  $d$  is the distance up the building. Simplifying, you get  $676 = 100 + d^2 \rightarrow 576 = d^2 \rightarrow 24 = d$ .

- 38. The correct answer is G.** Recall that the area of a square with side  $s$  is  $s^2$ . Finding the diameter of the circle, as shown below, it is clear that the side of the square is equal to the diameter of the circle, or  $2(5) = 10$ . Thus the area of the square is  $10^2$ , or 100 square feet.



- 39. The correct answer is C.** To find the length of the longest side of the second triangle, use ratios of corresponding sides of each triangle. For example,  $\frac{9}{7} = \frac{x}{13}$ , where  $x$  is the longest side of the second triangle. Cross-multiply to arrive at  $117 = 7x$ . Divide by 7 to get  $x = \text{about } 16.7$ .

If you selected answer choice B, the most common incorrect answer, you might have noticed that the difference in lengths of the smallest sides was 2 and then simply added 2 to the longest side of the first triangle to get 15 for the longest side of the second triangle.

- 40. The correct answer is H.** To find the measure of angle  $CDB$  in the figure, it is helpful to recognize that the sides  $BC$  and  $AD$  are parallel (definition of trapezoid) and are connected by the transversal  $BD$ . Angles  $CBD$  and  $ADB$  are alternate interior angles, and thus are equal and both measure  $25^\circ$ . Because  $A$ ,  $D$ , and  $E$  all lie along the same line, angle  $ADE = 180^\circ$ . Because angle  $ADE$  is made up of angles  $ADB$ ,  $CDB$ , and  $CDE$ , the measures of these three angles add up to  $180^\circ$ :  $25^\circ + CDB + 100^\circ = 180^\circ$ , thus the measure of angle  $CDB$  is  $55^\circ$ .

- 41. The correct answer is D.** This figure has 10 sides, but the lengths are given for only 7 sides. Those lengths add up to 36 inches. The perimeter is greater than this because of the missing 3 sides



so you can eliminate answer choices A and B. To solve this problem, use the information given to find the missing sides; based on the figure, you can see that the sum of right-facing sides equals the sum of left-facing sides, and the sum of top-facing sides equals the sum of bottom-facing sides. It is easy to see that the bottom-facing sides will equal the top-facing side, which has a length of 14. Since we have the values for all of the left-facing sides ( $5 + 4 + 3 = 12$ ), the right-facing sides also have the sum of 12. Thus the perimeter is  $14 + 14 + 12 + 12$ , or 52.

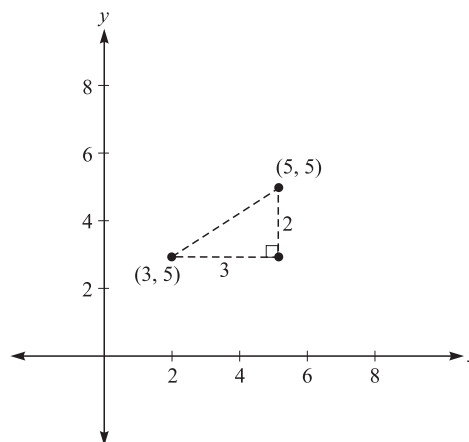
- 42. The correct answer is G.** To find out how many of the 517 seniors in Brighton High School are going to a state college, first find how many are going to college. You are given that  $\frac{4}{5}$  of the total number of graduating seniors (517) will be attending college:  $\frac{4}{5}$  of  $517 = 413.6$ , which can be rounded up to 414. Now, calculate the number of those 414 seniors who are going to a state college:  $\left(\frac{1}{2}\right)(414)$ , or about 207 seniors are going to a state college. This is closest to 200, answer choice G.

- 43. The correct answer is A.** You are given that  $x \boxtimes y = (x - 2y)^2$  and are asked to solve  $5 \boxtimes (-3)$ . To do this, simply replace  $x$  with 5, and  $y$  with  $-3$ , as follows:

$$\begin{aligned} x \boxtimes y &= (x - 2y)^2 \\ 5 \boxtimes (-3) &= (5 - 2(-3))^2 \\ 5 \boxtimes (-3) &= (5 - (-6))^2 \\ 5 \boxtimes (-3) &= (5 + 6)^2 \\ 5 \boxtimes (-3) &= (11)^2 \\ 5 \boxtimes (-3) &= 121 \end{aligned}$$

- 44. The correct answer is F.** Because 125% of “the number” is 425, then “the number” is  $425 \div 1.25$ , which equals 340. Calculate 65% of 340:  $340 \times 0.65 = 221$ .

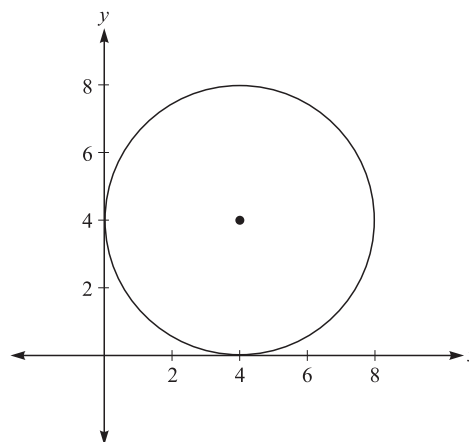
- 45. The correct answer is D.** To find the distance between 2 points in the standard  $(x, y)$  coordinate plane, use the distance formula, which states that  $d = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$ . Therefore, the distance is  $\sqrt{[(5 - 2)^2 + (5 - 3)^2]}$ , or  $(3^2 + 2^2)$ , which equals  $\sqrt{13}$ .



Another way to look at this problem would be to draw a picture as shown above. Then you can use the Pythagorean Theorem to find the hypotenuse of the triangle that is formed from the given points.

- 46. The correct answer is G.** To find the ratio of the circumference of 2 circles for which the ratio of their radii is 9:16, recognize that both circumference and radius are 1-dimensional attributes of a circle. Because of that, the ratios should be the same, 9:16. Another way is to use the ratio of the radii and let  $9x$  and  $16x$  be the radii of the two circles. Their circumferences would be  $2\pi(9x)$  and  $2\pi(16x)$ , respectively. When you put them in a ratio you see that the ratio  $2\pi(9x) : 2\pi(16x)$  simplifies to 9:16.

- 47. The correct answer is D.** The best approach to this question is to draw a diagram as shown below:



The equation of a circle is  $(x - h)^2 + (y - k)^2 = r^2$ . One way to find an equation for a circle is by using the coordinates of the center,  $(h, k)$ , and the radius,  $r$ . For this circle, the center is at  $(4, 4)$  and the radius is 4. Given center  $(4, 4)$  and radius 4, the circle has equation  $(x - 4)^2 + (y - 4)^2 = 4^2$ , or  $(x - 4)^2 + (y - 4)^2 = 16$ .

If you selected answer choice B, a common incorrect answer, you centered the circle at  $(0, 0)$ .

- 48. The correct answer is F.** To find an equivalent expression for  $\frac{2}{(1-i)} \times \frac{(1+i)}{(1+i)}$ , simply perform the calculations, as follows:

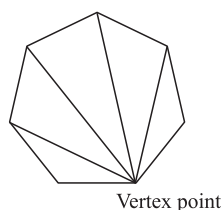
$$\begin{aligned}\frac{2(1+i)}{(1-i)(1+i)} &= \frac{2(1+i)}{(1-i^2)} \\ \frac{2(1+i)}{(1-i^2)} &= \frac{2(1+i)}{2} \\ \frac{2(1+i)}{2} &= 1+i\end{aligned}$$

- 49. The correct answer is D.** One approach to solving this problem is to make a table like the one below, showing the number of rows and the cumulative number of dots.

Row	1	2	3	4	5
Number of dots per row	2	4	6	8	10
Cumulative number of dots	2	2 + 4 = 6	6 + 6 = 12	8 + 12 = 20	10 + 20 = 30

The total number of dots in rows 1 and 2 is  $2(2+1)$ ; the total number of dots in row 3 is  $3(3+1)$ , and so on. You should be able to see that for the  $n$ th row, the total is the product of  $n$  and  $n+1$ , or  $n(n+1)$ .

- 50. The correct answer is J.** You are given that the total number of students is 24. If 21 students play basketball, and 9 students play soccer, there must be some overlap between basketball players and soccer players. The total number of students who play basketball and/or soccer is  $21 + 9$ , or 30; therefore,  $30 - 24$ , or 6 students must play both sports.
- 51. The correct answer is E.** To find the real numbers  $x$  such that  $x + 2 > x + 5$ , subtract  $x$  and 2 from both sides. The result is  $0 > 3$ , and because that inequality is never true, there is no solution for  $x$ . The solution set is the empty set.
- 52. The correct answer is H.** As shown below, there are 4 diagonals coming from each vertex point.

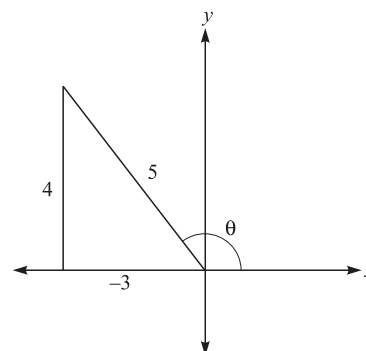


Because there are 7 vertex points, you might be tempted to conclude that there are  $7 \times 4$ , or 28 diagonals. But this method counts each diagonal exactly twice. Therefore, there are  $\frac{28}{2}$ , or 14 diagonals.

- 53. The correct answer is D.** You are given that 20% of John's friends selected vanilla ice cream as their favorite flavor. This means that 20% of the  $360^\circ$  in the circle will represent vanilla; 20% of 360 is equivalent to  $(0.20)(360^\circ)$ , or  $72^\circ$ . If you chose one of the other answers, you may have found the degree measure of any of the other flavors.

- 54. The correct answer is G.** One way to find  $\tan \theta$  given that  $\sin \theta = \frac{4}{5}$  and  $\frac{\pi}{2} < \theta < \pi$ , is to first find  $\cos \theta$ , then find  $\frac{\sin \theta}{\cos \theta}$ , which is equivalent to  $\tan \theta$ . To find  $\cos \theta$ , use the identity  $\sin^2 \theta + \cos^2 \theta = 1$  and the fact that  $\cos \theta < 1$  in Quadrant II ( $\frac{\pi}{2} < \theta < \pi$  would place the angle in Quadrant II). Use substitution to get  $\left(\frac{4}{5}\right)^2 + \cos^2 \theta = 1$ , or  $\frac{16}{25} + \cos^2 \theta = 1$ . After subtracting  $\frac{16}{25}$ , you get  $\cos^2 \theta = \frac{9}{25}$ . After taking the square root of both sides, you get  $\cos \theta = \pm \frac{3}{5}$ . Because  $\cos \theta < 1$  is in Quadrant II,  $\cos \theta = -\frac{3}{5}$ . Substitute this value into  $\frac{\sin \theta}{\cos \theta}$  to get  $\frac{(4/5)}{(-3/5)}$ , which equals  $-\frac{4}{3}$ .

Another way you could solve this problem would be to construct an angle in Quadrant II with  $\sin \theta = \frac{4}{5}$ , as shown below.



By virtue of the Pythagorean Theorem, the missing side of the right triangle is 3 units long, and is negative because it is along the negative side of the  $x$ -axis. From this triangle, knowing that tangent  $= \frac{\text{opposite}}{\text{adjacent}}$ , you can get  $\tan \theta = -\frac{4}{3}$ .

- 55. The correct answer is E.** To find the system of inequalities represented by the shaded region of the graph, first find the equations of the line through  $(-1, 0)$  and  $(0, 1)$  and the line through  $(-2, 0)$  and  $(0, -3)$ . These are  $y = x + 1$  (the  $y$ -intercept is 1) and  $y = \left(-\frac{3}{2}\right)x - 3$  (the  $y$ -intercept is  $-3$ ), respectively. Pay attention to the coordinating conjunctions, *and/or*.

- 56. The correct answer is K.** To find  $f(x + h)$  when  $f(x) = 2x^2 + 3$ , substitute  $(x + h)$  for  $x$  in  $f(x) = 2x^2 + 3$ , as follows:

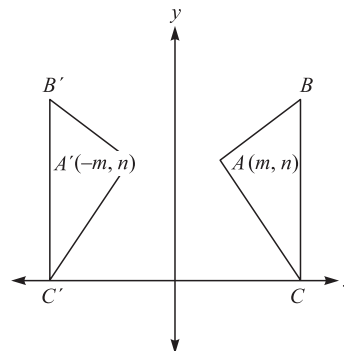
$$f(x + h) = 2(x + h)^2 + 3$$

$$2(x + h)^2 = 2(x^2 + 2xh + h^2) + 3$$

$$2(x^2 + 2xh + h^2) = 2x^2 + 4xh + 2h^2 + 3$$

- 57. The correct answer is D.** The equation  $y = \frac{x^2 + 3x}{x}$  can be simplified to  $y = \frac{x(x + 3)}{x}$ . Therefore, the graph of this seemingly complicated equation actually looks like a line, not a parabola, so eliminate answer choices A and B. This is equivalent to  $y = x + 3$  except when  $x = 0$ . When  $x = 0$ , the original equation is undefined. So the correct graph is  $y = x + 3$ , with a point removed where  $x = 0$ .

- 58. The correct answer is F.** To find the coordinates of vertex A after it is reflected across the  $y$ -axis, remember that a reflection across the  $y$ -axis does not change the sign of the  $y$ -coordinate but does change the sign of the  $x$ -coordinate. Therefore, you can eliminate answer choices G, H, and J. You might sketch a figure like the one below.



The reflection of  $A(m, n)$  across the  $x$ -axis is  $A'(-m, n)$ . The most popular incorrect answer is J, which gives the reflection of A over the line  $y = x$ .

- 59. The correct answer is B.** To obtain an expression for  $y$  in terms of  $x$  when  $x = 3r - 4$  and  $y = 3r + 2$ , first solve  $x = 3r - 4$  for  $r$  as follows:

$$x = 3r - 4$$

$$x + 4 = 3r$$

$$\frac{x + 4}{3} = r$$

Substitute that expression for  $r$  into  $y = 3r + 2$ , and solve for  $y$ :

$$y = 3\left[\frac{x + 4}{3}\right] + 2, \text{ which simplifies to } y =$$

$$(x + 4) + 2, \text{ or } y = x + 6.$$

- 60. The correct answer is H.** To find  $\cos \frac{\pi}{12}$  using  $\cos(\alpha - \beta) = (\cos \alpha)(\cos \beta) + (\sin \alpha)(\sin \beta)$  given that  $\frac{\pi}{12} = \frac{\pi}{3} - \frac{\pi}{4}$ , you can first substitute  $\frac{\pi}{3}$  for  $\alpha$  and  $\frac{\pi}{4}$  for  $\beta$  and get  $\cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = \left(\cos \frac{\pi}{3}\right)\left(\cos \frac{\pi}{4}\right) + \left(\sin \frac{\pi}{3}\right)\left(\sin \frac{\pi}{4}\right)$ . Using the table of values to substitute into that equation, you get  $\cos \frac{\pi}{12} = \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$ , or  $\frac{(\sqrt{6} + \sqrt{2})}{4}$ .